

## 2004 Paper 7 Question 8

### Information Theory and Coding

(a) Consider an alphabet of 8 symbols whose probabilities are as follows:

|               |               |               |                |                |                |                 |                 |
|---------------|---------------|---------------|----------------|----------------|----------------|-----------------|-----------------|
| A             | B             | C             | D              | E              | F              | G               | H               |
| $\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{1}{8}$ | $\frac{1}{16}$ | $\frac{1}{32}$ | $\frac{1}{64}$ | $\frac{1}{128}$ | $\frac{1}{128}$ |

- (i) If someone has selected one of these symbols and you need to discover which symbol it is by asking “yes/no” questions that will be truthfully answered, what would be the most efficient sequence of such questions that you could ask in order to discover the selected symbol? [2 marks]
- (ii) By what principle can you claim that each of your proposed questions is maximally informative? [2 marks]
- (iii) On average, how many such questions will need to be asked before the selected symbol is discovered? [2 marks]
- (iv) What is the entropy of the above symbol set? [2 marks]
- (v) Construct a uniquely decodable prefix code for the symbol set, and explain why it is uniquely decodable and why it has the prefix property. [2 marks]
- (vi) Relate the bits in your prefix code to the “yes/no” questions that you proposed in (i). [2 marks]
- (b) Explain the meaning of “self-Fourier”, and cite at least *two* examples of mathematical objects having this property. [3 marks]
- (c) Explain briefly:
- (i) sensation limit; [1 mark]
- (ii) critical band; [1 mark]
- (iii) Bark scale. [1 mark]
- (d) Which different aspects of perception do Weber’s law and Steven’s law model? [2 marks]