

# 2004 Paper 6 Question 11

## Semantics of Programming Languages

L1 has the expression syntax

$$e ::= n \mid b \mid e_1 \text{ op } e_2 \mid \mathbf{if } e_1 \mathbf{ then } e_2 \mathbf{ else } e_3 \\ \mid \ell := e \mid !\ell \mid \mathbf{skip} \mid e_1; e_2 \mid \mathbf{while } e_1 \mathbf{ do } e_2$$

(a) Give the reduction rules for conditionals and while-loops. [3 marks]

(b) Define *semantic equivalence*,  $e_1 \simeq_{\Gamma}^T e_2$ , for L1. [4 marks]

(c) For each of the following pairs, state whether they are semantically equivalent; if not, state a nontrivial condition on the subexpressions  $e, e_1, e_2, e_3$  that makes them so, and explain informally why it suffices.

(i)  $l := 3; e \stackrel{?}{\simeq} e; l := 3$  [3 marks]

(ii)  $e; (\mathbf{if } e_1 \mathbf{ then } e_2 \mathbf{ else } e_3) \stackrel{?}{\simeq} \mathbf{if } e_1 \mathbf{ then } e; e_2 \mathbf{ else } e; e_3$  [3 marks]

(iii)  $e; (\mathbf{if } e_1 \mathbf{ then } e_2 \mathbf{ else } e_3) \stackrel{?}{\simeq} \mathbf{if } e; e_1 \mathbf{ then } e_2 \mathbf{ else } e_3$  [3 marks]

(iv)  $\mathbf{while } !l \geq 0 \mathbf{ do } (e_2; e_3)$

$\stackrel{?}{\simeq}$   
 $\mathbf{if } !l \geq 0 \mathbf{ then } e_2; (\mathbf{while } !l \geq 0 \mathbf{ do } (e_3; e_2)); e_3 \mathbf{ else skip}$

[4 marks]