2004 Paper 12 Question 12

Complexity Theory

Recall that a *simple path* in a graph is a path with no repeated nodes. Consider the following two decision problems:

- Given a graph $G = (V, E)$, a positive integer k, source $s \in V$ and a target $t \in V$, is there a simple path from s to t of length *at least* k?
- Given a graph $G = (V, E)$, a positive integer k, source $s \in V$ and a target $t \in V$, is there a simple path from s to t of length *at most* k?

One of these problems is known to be in P while the other one is known to be NP-complete.

(a) Which of the two problems is in P and which is NP-complete? [2 marks]

(b) Describe a polynomial time algorithm for the problem that is in P. [6 marks]

(c) Give a proof of NP-completeness for the problem that is NP-complete. You may assume the NP-completeness of any problem, such as *Hamiltonian Cycle*, mentioned in the lecture course. [12 marks]