2004 Paper 11 Question 9

Mathematics for Computation Theory

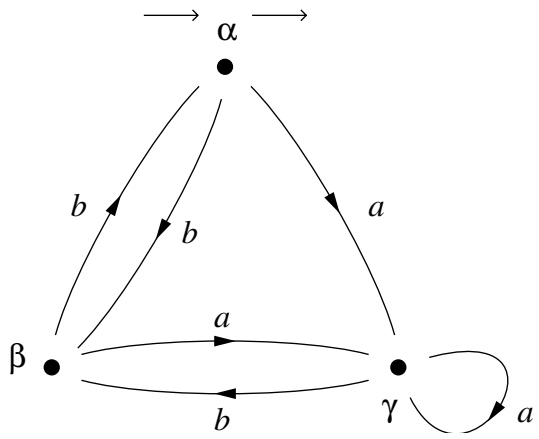
(a) Prove Arden's Rule for events, that $X = A^*B$ is the least solution of the inequality $X \geqslant B + AX$. [6 marks]

(b) Let $M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$ be a (2×2) event matrix. Show that the matrix

$$Y = \begin{pmatrix} (A + BD^*C)^* & A^*B(D + CA^*B)^* \\ D^*C(A + BD^*C)^* & (D + CA^*B)^* \end{pmatrix}$$

satisfies the equation $Y = I + MY$. [4 marks]

(c) The deterministic finite automaton M has a 2-symbol alphabet $\{a, b\}$, and a single accepting state α, the initial state. The transition diagram is as follows:



Show that the event accepted by M can be denoted by the regular expression

$$[a^*b(a\ a^*b)^*b]^*$$

[10 marks]