# 2004 Paper 10 Question 8

## Mathematics for Computation Theory

Let $A, B, C$ be sets. Define the *Cartesian product* $(A \times B)$ and the *disjoint union* $(A + B)$. [3 marks]

Let $f \subseteq (A \times B), g \subseteq (B \times C)$ be relations between $A$ and $B$, $B$ and $C$ respectively. Define the *inverse relation* $f^{-1}$ between $B$ and $A$ and the *product relation* $(f \circ g)$ between $A$ and $C$. [3 marks]

What conditions must be satisfied for the relation $f$ to be a function $f : A \to B$? [2 marks]

Write $(A \to B)$ for the set of all functions from $A$ to $B$. If $A, B$ are both *finite*, $|A| = a, |B| = b$, how many elements are there in $(A \times B),\ (A + B),\ (A \to B)$? [2 marks]

If $f$ and $f^{-1}$ are both functions, we say that $f$ is a *bijection*, and we write $A \cong B$. If $A, B$ are both finite and $f : A \to B$ is a bijection, prove that $a = b$. $(\star)$ [2 marks]

Establish explicit bijections between the following pairs of sets:

$(a)$ $A \to (B \times C)$, $\qquad (A \to B) \times (A \to C)$; [3 marks]

$(b)$ $(A + B) \to C$, $\qquad (A \to C) \times (B \to C)$. [4 marks]

If $A$, $B$, $C$ are all finite, verify that the cardinality condition $(\star)$ above is satisfied in each case. [1 mark]