

2003 Paper 8 Question 11

Numerical Analysis II

- (a) Let n_+ be the number of positive real roots of a polynomial $p_n(x)$. Let c be the number of changes of sign when the coefficients are taken in order. State *Descartes' rule of signs*. [2 marks]
- (b) If $p_3(x) = x^3 + 13x^2 + 54x + 72$ what does the rule say about the polynomials $p_3(x)$, $p_3(-x)$? [2 marks]
- (c) Using Descartes' rule, a simple search technique and factorisation, find all the real and complex roots of

$$q_5(x) = x^5 + 5x^4 + 32x^3 + 160x^2 + 256x + 1280.$$

[7 marks]

- (d) The Newton–Raphson formula $\tilde{x} = x - f(x)/f'(x)$ can often be used to find real roots of a polynomial. However, a user of numerical software reports failure to find an accurate root of

$$f(x) = 3x^4 - 28x^3 + 24x^2 + 144x + 432$$

for $x > 3$ using Newton–Raphson. The user has tried different starting values but gets either a floating-point exception or failure to converge. Using Descartes' rule on $f(x)$ and $f'(x)$ investigate the cause of these numerical problems. [9 marks]