COMPUTER SCIENCE TRIPOS Part IB

Tuesday 3 June 2003 1.30 to 4.30

Paper 4

Answer **five** *questions.*

Submit the answers in five **separate** *bundles, each with its own cover sheet. On each cover sheet, write the numbers of* **all** *attempted questions, and circle the number of the question attached.*

> **You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator**

1 Compiler Construction

A programming language has commands C and expressions E which may involve the terminals I (identifiers) and N (integer constants). Its grammar $\mathcal{G}$ (with start symbol S) is given by

$$S ::= C \ \texttt{eof}$$
$$C ::= I = E \mid \texttt{if } E \texttt{ then } C \mid \texttt{if } E \texttt{ then } C \texttt{ else } C$$
$$E ::= I \mid N \mid E + I$$

Construct

(a) a recursive descent parser, and [8 marks]

(b) the characteristic finite state machine (CFSM) of a LR(k), SLR(k) or LALR(k) parser, [12 marks]

explaining carefully whether there are any problems in the grammar $\mathcal{G}$ (and if so how you resolved them) and in adapting the grammar for recursive descent parsing and in adopting the CFSM so as to be suitable for LR(k), SLR(k) or LALR(k) parsing. It is not necessary to consider how the corresponding parse tree is constructed.

2 Economics and Law

(a) Describe *three* ways in which progress in information technology affects the expected size of firms, including at least one factor that favours larger firms and at least one factor that favours smaller ones. [12 marks]

(b) If the US Department of Justice had managed to split Microsoft into an operating systems company and an applications company, do you think the demerged companies would have been worth more, or less, in the long run? Briefly justify your answer. [4 marks]

(c) Would such a demerger have had an effect on the size of the average software company, and if so, what? [4 marks]

3 Data Structures and Algorithms

An application requires a hash table to hold up to 10^6 key–value pairs where both the keys and the values are integers. Carefully describe the two possible implementations: (*a*) *open hashing* using linked lists of key–value pairs outside the table and (*b*) *closed hashing* in which all key–value pairs are held within the hash table. [10 marks]

Discuss how you would decide which method is most suitable for the given application. [4 marks]

Assuming the table has exactly 10^6 entries and that keys, values and list pointers are all of size 4 bytes, and that you have allocated a total of 16 million bytes for the table (and hash chains), estimate the expected number of key comparisons necessary to locate an existing entry for (*a*) the open hash table and (*b*) the closed hash table. [6 marks]

4 Comparative Programming Languages

Give a brief description of the main syntactic constructs used in Smalltalk (or Squeak) programs, illustrating your answer by explaining the meaning of the following fragment of code:

```
[self isAwake]
whileTrue:
[| item |
  item := self askForCookie.
  (self isCookie: item)
    ifTrue: [self eat: item]
    ifFalse: [self complainAbout:item].
    (self isFull) ifTrue: [self sleep]]
```
[10 marks]

Suggest how you implement in Smalltalk (or Squeak) a binary tree in which each node contains an integer and pointers to two or fewer other nodes of the same kind. [4 marks]

Outline the code you would use (*a*) to construct this kind of tree, and (*b*) to sum all the integers in a given tree. [6 marks]

5 Operating Systems II

(*a*) Describe the basic operation of a *log-structured file system*. What are the potential benefits? What are the problems? [8 marks]

(*b*) Several modern file systems make use of *journalling*. Describe how a journal is used by the file system, and the situations in which it is beneficial. [6 marks]

(*c*) You are assigned the task of designing a tape backup strategy for an important file server. The goal is to maximise the time any file is held on backup while minimising the number of tapes required. Sketch your strategy, commenting on the support you require from the file system, and justifying your design decisions. [6 marks]

6 Continuous Mathematics

(*a*) State the definition of the Fourier Transform, $F(\mu)$, of a function $f(x)$ and give the expression for the inverse Fourier Transform of $f(x)$ in terms of $F(\mu)$. [4 marks]

(*b*) Consider the function

$$f(x) = \begin{cases} e^{-ax} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

for $a > 0$ and find its Fourier Transform, $F(\mu)$. [4 marks]

(*c*) Now consider the function

$$f(x) = e^{-a|x|}$$

for $a > 0$ where $-\infty < x < \infty$ and find its Fourier Transform. [4 marks]

(*d*) Show that the Fourier Transform of the function

$$f(x) = \frac{1}{(1 + x^2)}$$

is $F(\mu) = \frac{1}{2}e^{-|\mu|}$. [8 marks]

7 Numerical Analysis I

(a) Explain briefly the *back substitution* algorithm for solving an upper triangular system of linear equations. Why is this important? What is *forward substitution*? [5 marks]

(b) What is meant by a *symmetric positive definite* matrix? [2 marks]

(c) Given that $A = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$ is positive definite and

$$A = \begin{pmatrix} 1 & \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ & 1 \end{pmatrix}$$

show how this factorisation may be used to solve the equations

$$A \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}.$$

[6 marks]

(d) Now consider the equations

$$\begin{pmatrix} 3 & 4 & 1 \\ 0 & 8 & 2 \\ 3 & 2 & 5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 16 \\ 14 \\ 8 \end{pmatrix}.$$

Pre-multiply each side by $\begin{pmatrix} 1 & 0 & 0 \\ 4 & -1 & -4 \\ 1 & 0 & -1 \end{pmatrix}$ and hence find the solution.

[7 marks]

[TURN OVER

8 Concurrent Systems and Applications

For each of the following topics explain what features Java provides and give a brief example of when you might use them:

(*a*) scheduling clean-up operations when a data structure ceases to be reachable;

(*b*) calling a method given its name as a string;

(*c*) converting a data structure based on objects to one held in a `byte[]` array;

(*d*) creating and instantiating a new class given its definition in a `byte[]` array;

(*e*) implementing a method in a language other than Java.

There is no need to recall the precise names and parameters of library facilities. So long as you are clear about the principles involved, you may assume that your reader will be able to look up the details.   [4 marks each]

9 Computation Theory

What is the *Church–Turing Thesis*? Briefly describe some evidence that it is true.
[4 marks]

Using the Church–Turing Thesis, or otherwise, show that if $f(x)$ and $g(x)$ are partial recursive functions of a single argument, then so are the following functions, where *dom(f)* denotes the set of integers x for which $f(x)$ is defined, and similarly for *dom(g)*.

$$h(x) = \begin{cases} x & \text{if } x \in dom(f) \text{ and } x \in dom(g) \\ \text{undefined} & \text{otherwise} \end{cases}$$ [4 marks]

$$k(x) = \begin{cases} x & \text{if } x \in dom(f) \text{ or } x \in dom(g) \\ \text{undefined} & \text{otherwise} \end{cases}$$ [6 marks]

Is the partial function defined by

$$f'(x) = \begin{cases} x & \text{if } x \notin dom(f) \\ \text{undefined} & \text{otherwise} \end{cases}$$

necessarily partial recursive if f is? Justify your answer.   [6 marks]

END OF PAPER