

2002 Paper 8 Question 1

Denotational Semantics

Let D be a cpo with bottom element \perp . Let $k : D \rightarrow D$ be a continuous function. Let $\mathbb{B} = \{true, false\}$. Define the conditional function

$$\text{if} : \mathbb{B}_{\perp} \times D \times D \rightarrow D$$

by $\text{if}(b, d, d') = d$ if $b = true$, d' if $b = false$, and \perp otherwise. Let $h : D \rightarrow \mathbb{B}_{\perp}$ be a continuous function which is strict (so $h(\perp) = \perp$).

The function f^* is the least continuous function from D to D such that

$$\forall x \in D. f^*(x) = \text{if}(h(x), x, f^*(k(x))) .$$

(a) State the principle of fixed point induction. [3 marks]

(b) Show that

$$\forall x \in D. h(f^*(x)) = \text{if}(h(x), h(x), h(f^*(k(x)))) . \quad [4 \text{ marks}]$$

(c) Prove that the property

$$Q(f) \Leftrightarrow_{def} \forall x \in D. h(f(x)) \sqsubseteq true$$

is admissible. [5 marks]

(d) Prove $Q(f^*)$ by fixed point induction. [8 marks]