2002 Paper 5 Question 12

Complexity Theory

(*a*) Give a precise definition of what is meant by the statement that a decision problem A is *polynomial-time reducible* to a decision problem B. [2 marks]

(*b*) Consider the following three decision problems on graphs.

Connect—the collection of graphs G such that there is a path between any two vertices of G.

Hamilton—the collection of graphs that contain a Hamiltonian cycle.

non-3-colour—the collection of graphs that are not 3-colourable.

For each of the following statements, state whether it is true, false or an unresolved open question. Give a brief justification for your answer.

(*i*) **Connect** is decidable by a polynomial time algorithm.

(*ii*) **Hamilton** is decidable by a polynomial time algorithm.

(*iii*) **non-3-colour** is decidable by a polynomial time algorithm.

(*iv*) **Connect** is polynomial-time reducible to **Hamilton**.

(*v*) **Hamilton** is polynomial-time reducible to **non-3-colour**.

(*vi*) **non-3-colour** is polynomial-time reducible to **Connect**.

[3 marks each]