## 2002 Paper 5 Question 11

## Logic and Proof

(a) For each of the following formulae, state (with justification) whether it is satisfiable, valid or neither:

$$
\begin{array}{cc}
((Q \rightarrow R) \rightarrow Q) \wedge \neg Q & {[2 \text { marks }]} \\
((P \leftrightarrow Q) \leftrightarrow P) \leftrightarrow Q & {[2 \text { marks }]} \\
\exists x y[P(x, y) \rightarrow \forall x y P(x, y)] & {[3 \text { marks }]} \\
{[\forall x(P(x) \rightarrow Q(x)) \wedge \exists x P(x)] \rightarrow \forall x Q(x)} & {[3 \text { marks }]}
\end{array}
$$

(b) Briefly outline the semantics of first-order logic, taking as an example the formula $\forall x y f(x, y)=f(y, x)$.
(c) Exhibit a model that satisfies both of the following formulae ( $a$ is a constant):

$$
\begin{gathered}
\forall x g(x) \neq a \\
\forall x y[g(x)=g(y) \rightarrow x=y]
\end{gathered}
$$

