2002 Paper 5 Question 11

Logic and Proof

- (a) For each of the following formulae, state (with justification) whether it is satisfiable, valid or neither:
 - $((Q \to R) \to Q) \land \neg Q$ [2 marks]

$$((P \leftrightarrow Q) \leftrightarrow P) \leftrightarrow Q \qquad [2 \text{ marks}]$$

$$\exists xy \left[P(x, y) \to \forall xy P(x, y) \right]$$
 [3 marks]

$$[\forall x (P(x) \to Q(x)) \land \exists x P(x)] \to \forall x Q(x)$$
 [3 marks]

- (b) Briefly outline the semantics of first-order logic, taking as an example the formula $\forall xy f(x, y) = f(y, x)$. [6 marks]
- (c) Exhibit a model that satisfies both of the following formulae (a is a constant):

$$\forall x \, g(x) \neq a$$
$$\forall xy \, [g(x) = g(y) \rightarrow x = y]$$

[4 marks]