

2002 Paper 4 Question 6

Computation Theory

- (a) Your mathematician friend can prove to you that there are uncountably many functions from numbers to numbers, but does not know any computation theory. Explain to her what is meant by a *partial recursive* function and by a (total) *recursive* function. How would you convince her that there must exist functions that are not recursive? [12 marks]
- (b) What does it mean for a set of numbers $S \subseteq \mathbb{N}$ to be
- (i) *decidable*; [1 mark]
 - (ii) *recursively enumerable* (r.e.)? [2 marks]
- (c) S is called *co-r.e.* if its complement $\{x \in \mathbb{N} \mid x \notin S\}$ is r.e. Show that S is decidable if it is both r.e. and co-r.e. (Any standard results about computable functions that you use should be clearly stated.) [5 marks]