2002 Paper 2 Question 5

Probability

An engineer has been monitoring the performance of two communication channels and has established that, on average, channel A sustains λ_A faults each month and channel B sustains λ_B faults each month. In each case a Poisson distribution may be assumed. It may also be assumed that the channels are independent.

(a) Let X and Y be random variables whose values, r and s, are the numbers of faults each month on channel A and channel B respectively. Show that the derived random variable $X + Y$ is also Poisson distributed and determine the associated parameter. [6 marks]

(b) Let $n = r + s$, the total number of faults in a given month. For given n, the engineer notes that any number from 0 to all n faults may be attributable to channel A and assumes that this number is Binomially distributed. Explain, informally, why this is a reasonable assumption. [4 marks]

(c) Noting the result of part (a), derive the parameters of the Binomial distribution which governs the random variable X *given that the total number of faults is* n. [8 marks]

(d) Supposing that $\lambda_A = 4$ and $\lambda_B = 6$, what is the expected number of faults attributable to channel A if, one month, 5 faults were recorded in total? [2 marks]