COMPUTER SCIENCE TRIPOS Part IB

Tuesday 4 June 2002 1.30 to 4.30

Paper 4

Answer **five** *questions.*

Submit the answers in five **separate** *bundles, each with its own cover sheet. On each cover sheet, write the numbers of* **all** *attempted questions, and circle the number of the question attached.*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

1 Concurrent Systems and Applications

Below are four potential problems and two proposed solutions for each one. For each of the problems, give a brief example showing the proposed solutions and explain the advantages and disadvantages of each.

(*a*) Data held in one object is to be made available throughout a large, possibly distributed, application.

(*i*) Store the data in a field with the `public` modifier.

(*ii*) Store it in a field with the `private` modifier but provide `public` methods to access its value.

(*b*) A class `C` implements an interface `I1` but some code is designed to access it through an alternative interface `I2`. The two interfaces support similar operations.

(*i*) Define a new class using inheritance.

(*ii*) Use the *Adapter* design pattern.

(*c*) You are designing a data structure and need to decide how to perform concurrency control in case it is used in a multi-threaded application.

(*i*) Use `synchronized` methods (or other features) to make the methods safe for concurrent use.

(*ii*) Do not manage concurrency here and add comments to the source code.

(*d*) You have a class that defines how to communicate with a remote server using a TCP socket. The connection is established in the constructor and you must decide how to close it.

(*i*) Provide an explicit `close` method in your class.

(*ii*) Use a `finalize` method.

[5 marks each]

2 Compiler Construction

The specification for a pocket-calculator-style programming language is as follows:

- Valid inputs consist either of an Expression followed by the `enter` button or of an Expression followed by `store` Identifier `enter` ;
- Expressions consist of Numbers and Identifiers connected with the binary operators $\boxed{+}$, $\boxed{\times}$ and $\boxed{\uparrow}$ (in increasing binding power), with the unary operators $\boxed{-}$ and `abs`, and possibly grouped with parentheses. Unary operators bind more strongly than $\boxed{+}$ but weaker than $\boxed{\times}$ so that $-a + b$ means $(-a) + b$ but $-a \times b$ means $-(a \times b)$;
- Numbers consist of a sequence of at least one digit, possibly interspersed with exactly one decimal point, and possibly followed by an exponential marker "`e`" followed by a signed integer, e.g. `6.023e+22`. Identifiers are sequences of lower-case letters.

(*a*) Give a Context-Free Grammar (Type 2 in the Chomsky Hierarchy) for the set of valid input sequences using names beginning with an upper-case letter for non-terminals. It should be complete in that you should go as far as to define e.g.

```
Letter ::= a | b | ... | z
```

[10 marks]

(*b*) Indicate, giving brief reasoning, which non-terminals are appropriate to be processed using lexical analysis and which using syntax analysis proper. [5 marks]

(*c*) Give yacc or CUP input describing those elements deemed in part (*b*) to be suitable for syntax analysis. You need not give "semantic actions". [5 marks]

3 ECAD

A naïve Verilog programmer has written the following code in an attempt to implement an up–down counter. The correct behaviour should be:

- The counter should increment once each time the `up` button is pressed.
- The counter should decrement once each time the `down` button is pressed.
- The `error` flag should be set if both buttons are pressed at the same time.
- The `error` flag and `counter` should be set to zero when `reset` is high.
- The counter should be 8 bits in length.
- The input buttons are asynchronous and supply "1" when pressed, "0" when not pressed.

```
module UpDownCounter(clk, reset, up, down, count, error)
  input  clk;      // input clock
  input  reset;    // reset
  input  up;       // from the up button
  input  down;     // from the down button
  output counter;  // output the count value
  output error;    // error flag (1=error, 0=no error)

  reg [7:0] counter;
  always @(posedge clk) {
    if(reset) {
      counter <= 0;
      error   <= 0;
    }
    if(up)   counter <= counter + 1;
    if(down) counter <= counter - 1;
    if(up && down) error <= 1;
  }
endmodule
```

(*a*) Explain what is wrong with the above Verilog code. [10 marks]

(*b*) Write a corrected and commented version of the `UpDownCounter` module including any ancillary modules. [10 marks]

4 Data Structures and Algorithms

(*a*) Carefully describe how Shellsort works and state an estimate of its efficiency using big O notation. [8 marks]

(*b*) Robert Sedgewick suggests that a good sequence of separations used in the algorithm is $\ldots, 121, 40, 13, 4, 1$. Explain why this is a good sequence. Under what circumstances would you recommend a sequence that approaches 1 more rapidly? [4 marks]

(*c*) Describe how radix sort from the least significant end works and suggest a data structure that could be used in its implementation. [8 marks]

5 Continuous Mathematics

Suppose that $F(\mu)$ is the Fourier transform of the function $f(x)$.

(*a*) State the integral expression for $F(\mu)$ in terms of $f(x)$ and the inverse transform for $f(x)$ in terms of $F(\mu)$. [2 marks]

(*b*) Determine the *shift rule* for the Fourier transform of $f(x-\alpha)$ where α is a constant. [3 marks]

(*c*) Determine the *scale rule* for the Fourier transform of $f(\alpha x)$ where α is a non-zero constant. [3 marks]

(*d*) Given that the standard normal density function $\frac{1}{\sqrt{2\pi}}e^{-x^2/2}$ has Fourier transform $F(\mu)$ use the scale and shift rules to determine the Fourier transform of the normal density $\frac{1}{\sqrt{2\pi}\sigma}e^{-(x-\gamma)^2/(2\sigma^2)}$. Here γ is a real number and σ is a positive real number. [6 marks]

(*e*) Show that

$$G(\mu) = A\int_{-\infty}^{\infty} g(x)e^{-ia\mu x}\,dx$$

implies that

$$g(x) = \frac{|a|}{2\pi A}\int_{-\infty}^{\infty} G(\mu)e^{ia\mu x}\,d\mu$$

for all non-zero constants a and A. You may assume that the result holds in the special case when $a = 1$ and $A^{-1} = 2\pi$. [6 marks]

6 Computation Theory

(*a*) Your mathematician friend can prove to you that there are uncountably many functions from numbers to numbers, but does not know any computation theory. Explain to her what is meant by a *partial recursive* function and by a (total) *recursive* function. How would you convince her that there must exist functions that are not recursive? [12 marks]

(*b*) What does it mean for a set of numbers $S \subseteq \mathbb{N}$ to be

(*i*) *decidable*; [1 mark]

(*ii*) *recursively enumerable* (r.e.)? [2 marks]

(*c*) S is called *co-r.e.* if its complement $\{x \in \mathbb{N} \mid x \notin S\}$ is r.e. Show that S is decidable if it is both r.e. and co-r.e. (Any standard results about computable functions that you use should be clearly stated.) [5 marks]

7 Numerical Analysis I

(*a*) Define *absolute error* and *relative error*. How are they related? How are absolute errors combined when two numbers are added together? How are relative errors combined when two numbers are multiplied together? [5 marks]

(*b*) Explain the term *loss of significance* in terms of absolute error and relative error. [1 mark]

(*c*) Writing δ_x for the relative error in x, what is the worst-case *relative error* in evaluating x^2? What is the worst-case *absolute error*? What is the worst-case *absolute error* in evaluating $x^2 - y^2$? [3 marks]

(*d*) Let δ_s, δ_c be the relative errors in the values of $\sin\theta$, $\cos\theta$ respectively. Find the worst-case relative errors in evaluating each of the formulae

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos 2\theta = 2\cos^2\theta - 1$$

For what values of $\cos\theta$ does the second formula display loss of significance? [6 marks]

(*e*) Consider the evaluation of $x^2 + y^2 - z^2$ in two cases

(*i*) $|y| \simeq |z|$, $|x|$ is very small,

(*ii*) $|x| \simeq |y| \simeq |z|$, $|x|$ is not small.

Taking each case separately, can loss of significance occur? Explain your answers. [4 marks]

(*f*) How would you compute $x^2 + y^2 - z^2$ to achieve greater accuracy, especially if guard digits were in use? [1 mark]

8 Computer Graphics and Image Processing

(*a*) With reference to the characteristics and performance of the human visual system, provide an estimate for each of the following. In each case you are expected to justify your estimate:

(*i*) the maximum resolution required by a display device;

(*ii*) the maximum number of distinct intensity levels required by a display device;

(*iii*) the optimal number of dimensions required to represent colour;

(*iv*) the maximum refresh rate required of a CRT monitor.

[5 marks]

(*b*) A programmer suggests three different implementations of a polygon drawing algorithm:

(*i*) standard z-buffer;

(*ii*) standard A-buffer with an 8×8 mask size;

(*iii*) standard z-buffer at 8×8 normal resolution followed by averaging operation which produces a normal resolution image by finding the average value of each 8×8 block.

Compare the three implementations in terms of both execution speed and resulting image quality. Which would be the best implementation to use if the average polygon covers 50 pixels? Which would be the best implementation to use if the average polygon covers 2 pixels? Which would be the best implementation to use if the display resolution was equal to the answer that you gave to (*a*)(*i*) above? [7 marks]

(*c*) Show how to perform 2D rotation about an arbitrary point. Provide a matrix in homogeneous coordinates for each step in the operation. [2 marks]

(*d*) Show how to perform 3D rotation about an arbitrary axis. Again, give matrices in homogeneous coordinates for each step in the operation. [6 marks]

END OF PAPER