Logic and Proof

(a) In the context of clause-based proof methods, define the notion of *pure literal* and describe what should be done if the set of clauses contains pure literals.

[3 marks]

(b) Use the Davis–Putnam method to discover whether the following set of clauses is satisfiable. If they are satisfiable, show a satisfying interpretation.

 $\{P,R\} \qquad \{\neg P,\neg R\} \qquad \{P,\neg Q\} \qquad \{\neg Q,R\} \qquad \{\neg P,Q,R\}$

[6 marks]

- (c) The three-fingered inhabitants of the planet Triterra build base-3 computers. A Triterran named Randal Tryant has found a way of verifying base-3 combinational logic. His Ordered Ternary Decision Diagrams (OTDDs) are the same as a technology used on planet Earth except that all variables and expressions range over the values 0, 1 and 2 instead of just 0 and 1.
 - (i) Describe how a full ternary decision tree can be reduced to an OTDD without regard for efficiency. [2 marks]
 - (*ii*) Sketch an efficient algorithm to convert a ternary expression directly to an OTDD without constructing the full decision tree. For a typical ternary connective use modulo-3 multiplication, written as \otimes . [6 marks]
 - (*iii*) Demonstrate your algorithm by applying it to the ternary expression $((i \otimes i) \otimes j) \otimes 2$. [3 marks]