2001 Paper 5 Question 11

Logic and Proof

- (a) Explain the meaning of the notation $A \models B$, where A and B denote formulae of (i) propositional logic and (ii) S4 modal logic. [2+6 marks]
- (b) For each of the following equivalences, state whether it holds or not, justifying each answer rigorously.

$$\begin{array}{lll} (P \land (Q \to R)) \to S &\simeq & (\neg P \lor \neg Q \lor S) \land (\neg P \lor \neg R \lor S) \\ (P \to Q) \to (Q \to P) &\simeq & (Q \to P) \\ \forall xy \left(P(x) \lor \neg P(y) \right) &\simeq & \forall xy \left(P(x) \leftrightarrow P(y) \right) \end{array}$$

[3+3+6 marks]