

## 2001 Paper 4 Question 9

### Numerical Analysis I

(a) What is meant by a *symmetric positive definite matrix*? [3 marks]

(b) Verify that  $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$  is positive definite. [4 marks]

(c) The Choleski factorisation  $\mathbf{A} = \mathbf{LDL}^T$  is to be applied to the solution of  $\mathbf{Ax} = \mathbf{b}$ , where  $\mathbf{b} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ . It is found that

$$\mathbf{L} = \begin{pmatrix} 1 & \\ \frac{1}{2} & 1 \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} 2 & \\ & \frac{3}{2} \end{pmatrix}.$$

The next step in the method is to solve  $\mathbf{Ly} = \mathbf{b}$  to get  $\mathbf{y} = \begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix}$ . Form the upper triangular system of equations needed to complete the solution. [4 marks]

(d) Solve these equations. [2 marks]

(e) What is meant by the *order of convergence* of an iterative process? [1 mark]

(f) State the Newton–Raphson formula for solving  $f(x) = 0$  for scalar  $x$ . What is the order of convergence of this method? [2 marks]

(g) This method is used to solve  $f(x) = x^2 - 4 = 0$  using IEEE Double Precision with a certain starting value  $x_0$ . It is found that the third iterate  $x_3 \simeq 2.0006$ , and  $x_4 \simeq 2.00000009$ . Very roughly, how many significant decimal digits of accuracy would you expect in  $x_5$ ? Explain your answer. [4 marks]