2001 Paper 11 Question 9

Numerical Analysis I

(a) What is meant by a *symmetric positive definite matrix?* [3 marks]

(b) Verify that $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ is positive definite. [4 marks]

(c) The Choleski factorisation $\mathbf{A} = \mathbf{LDL}^T$ is to be applied to the solution of $\mathbf{Ax} = \mathbf{b}$, where $\mathbf{b} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. It is found that

$$\mathbf{L} = \begin{pmatrix} 1 & \\ \frac{1}{2} & 1 \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} 2 & \\ & \frac{3}{2} \end{pmatrix}.$$

The next step in the method is to solve $\mathbf{Ly} = \mathbf{b}$ to get $\mathbf{y} = \begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix}$. Form the upper triangular system of equations needed to complete the solution. [4 marks]

(d) Solve these equations. [2 marks]

(e) What is meant by the *order of convergence* of an iterative process? [1 mark]

(f) State the Newton–Raphson formula for solving $f(x) = 0$ for scalar x. What is the order of convergence of this method? [2 marks]

(g) This method is used to solve $f(x) = x^2 - 4 = 0$ using IEEE Double Precision with a certain starting value x_0. It is found that the third iterate $x_3 \simeq 2.0006$, and $x_4 \simeq 2.00000009$. Very roughly, how many significant decimal digits of accuracy would you expect in x_5? Explain your answer. [4 marks]