2000 Paper 9 Question 11

Information Theory and Coding

(*a*) Prove that the information measure is additive: that the information gained from observing the combination of N independent events, whose probabilities are p_i for $i = 1 \ldots N$, is the *sum* of the information gained from observing each one of these events separately and in any order. [4 marks]

(*b*) What is the shortest possible code length, in bits per average symbol, that could be achieved for a six-letter alphabet whose symbols have the following probability distribution?

$$\left\{ \tfrac{1}{2}, \tfrac{1}{4}, \tfrac{1}{8}, \tfrac{1}{16}, \tfrac{1}{32}, \tfrac{1}{32} \right\}$$

[3 marks]

(*c*) Suppose that ravens are black with probability 0.6, that they are male with probability 0.5 and female with probability 0.5, but that male ravens are 3 times more likely to be black than are female ravens.

If you see a non-black raven, what is the probability that it is male? [4 marks]

How many bits worth of information are contained in a report that a non-black raven is male? [1 mark]

Rank-order for this problem, from greatest to least, the following uncertainties:

(*i*) uncertainty about colour
(*ii*) uncertainty about gender
(*iii*) uncertainty about colour, given only that a raven is male
(*iv*) uncertainty about gender, given only that a raven is non-black

[3 marks]

(*d*) If a continuous signal $f(t)$ is *modulated* by multiplying it with a complex exponential wave $\exp(i\omega t)$ whose frequency is ω, what happens to the Fourier spectrum of the signal?

Name a very important practical application of this principle, and explain why modulation is a useful operation.

How can the original Fourier spectrum later be recovered? [3 marks]

(*e*) Which part of the 2D Fourier Transform of an image, the amplitude spectrum or the phase spectrum, is indispensable in order for the image to be intelligible?

Describe a demonstration that proves this. [2 marks]