2000 Paper 6 Question 12

Complexity Theory

State the hierarchy theorems for time and space. [4 marks]

A linear time reduction from a language L_1 to L_2 is a reduction that can be computed by a deterministic Turing machine in time $O(n)$.

A class of languages $\mathcal{C}$ is closed under linear time reductions if whenever $L_2 \in \mathcal{C}$ and L_1 is linear-time reducible to L_2, then $L_1 \in \mathcal{C}$.

For *each* of the following complexity classes (a) to (d), say

- whether it is closed under linear time reductions

- whether it contains problems that are complete under linear time reductions

Give full justification for your answers.

(a) $\mathrm{DSPACE}(n^2)$ [4 marks]

(b) L [4 marks]

(c) P [4 marks]

(d) NP [4 marks]