2000 Paper 5 Question 12

Complexity Theory

Give precise definitions of *polynomial time reductions* and *NP-completeness*.

[2 marks each]

Consider the following two decision problems on *undirected* graphs.

> **3-node-colourability**: the collection of graphs $G = (V, E)$ for which there is a mapping $\chi : V \to \{r, g, b\}$ such that if $(u, v) \in E$, then $\chi(u) \neq \chi(v)$.
>
> **3-edge-colourability**: the collection of graphs $G = (V, E)$ for which there is a mapping $\chi : E \to \{r, g, b\}$ such that if $(u, v), (u, v') \in E$, with $v \neq v'$, then $\chi(u, v) \neq \chi(u, v')$.

Show that there is a polynomial time reduction from **3-edge-colourability** to **3-node-colourability**. [8 marks]

The problem **3-edge-colourability** is known to be NP-complete. Using this information, for *each* of the following statements, state whether or not it is true. In each case, give complete justification for your answer.

(*a*) There is a polynomial time reduction from **3-node-colourability** to **3-edge-colourability**. [3 marks]

(*b*) **3-node-colourability** is NP-complete. [3 marks]

(*c*) **3-edge-colourability** is in PSPACE. [2 marks]