2000 Paper 3 Question 8

Continuous Mathematics

(a) When numerically computing the solution to an ordinary differential equation (ODE) that involves higher-than first-order derivatives:

(i) What is to be done about the higher-than first-order terms, and how can this be accomplished? [4 marks]

(ii) Illustrate this step for the following ODE, in which functions $r(x)$ and $q(x)$ are known and we seek to compute the solution $y(x)$:

$$\frac{d^2y}{dx^2} + q(x)\frac{dy}{dx} = r(x)$$ [4 marks]

(b) (i) State the incrementing rule for the Euler method of numerical integration, in terms of:

- $f(x_n)$, the estimate of the solution $f(x)$ at the current point x_n
- $f(x_{n+1})$, the new estimate of $f(x)$ for the next point x_{n+1}
- the integration stepsize h, which is the interval $(x_{n+1} - x_n)$
- $f'(x_n)$, the expression given by the ODE for the derivative of the desired solution $f(x)$ at the current point x_n

[4 marks]

(ii) What might happen to your solution if the stepsize h is too large? [2 marks]

(iii) What might happen to your solution if you make the stepsize h too small? [2 marks]

(iv) What is the primary advantage of the Runge–Kutta method over the Euler method for numerical integration of ODEs? [2 marks]

(v) Under what conditions might you wish to make the stepsize h adaptive rather than fixed? How should you adapt it? [2 marks]