2000 Paper 11 Question 10

Numerical Analysis I

Define the *absolute error* ε_x and *relative error* δ_x in representing a number x. How are these errors related? Which type of error is associated with the term *loss of significance*? Define *machine epsilon* ε_m. [4 marks]

Write down expressions for the worst case errors $|\delta_{xy}|$ and $|\varepsilon_{xy}|$ in computing xy. Express your answers in terms of δ_x and δ_y. Hence find expressions for $|\varepsilon_{xy+z}|$ and $|\delta_{xy+z}|$ in terms of δ_x, δ_y and δ_z. [6 marks]

Assuming $|\,\delta_x\,| = |\,\delta_y\,| = |\,\delta_z\,| = \varepsilon_m$, find an expression for $|\,\delta_{xy+z}\,|/\varepsilon_m$. When would you expect loss of significance in computing $xy + z$? [3 marks]

The formula

$$\tilde{f}'(x) = \frac{f(x+h) - f(x)}{h}$$

is used to compute $f'(x)$ for a certain function $f(x)$ when $\varepsilon_m = 10^{-10}$. The formula is applied at $x = 0.2$ where $f(0.2) = 1.1$.

(a) If $h = 10^{-3}$ then $\tilde{f}'(0.2) \simeq 8.44$.

(b) If $h = 10^{-8}$ then $\tilde{f}'(0.2) \simeq 8.40$.

However, it is known that $f'(0.2) = 8.42$ to 3 significant digits. Why is (a) such a poor estimate? Why is (b) such a poor estimate? [4 marks]

Suggest a more suitable value for h. Roughly how many correct significant decimal digits would you expect to get in your answer? [3 marks]