1999 Paper 9 Question 6

Communicating Automata and Pi Calculus

Define the notion of *sorting* in the π-calculus, based upon a set $\mathcal{S}$ of non-parametric sorts. What is meant by saying that a set of agents *respects* a sorting? Explain briefly why, if $P \xrightarrow{\tau} P'$, then P' respects any sorting respected by P. [5 marks]

Simple data can be represented as abstractions in the π-calculus. In particular, if *True* and *False* represent the truth-values, then $\mathit{True}\langle b\rangle$ and $\mathit{False}\langle b\rangle$ are processes representing each truth-value located at b. Define these abstractions. Also, for arbitrary processes P and Q, define an abstraction $\mathit{Cond}(P, Q)$ such that

$$\begin{array}{lll} \mathit{Cond}(P,Q)\langle b\rangle \mid \mathit{True}\langle b\rangle & \rightarrow^* & P \\ \mathit{Cond}(P,Q)\langle b\rangle \mid \mathit{False}\langle b\rangle & \rightarrow^* & Q \end{array}$$

and demonstrate these reactions. Give a sorting respected by these agents. [6 marks]

Define π-calculus abstractions which may be used to represent lists, whose elements may in turn be represented by abstractions. By analogy with *Cond*, define an abstraction $\mathit{Listcases}(P, (v\ell)Q)$ which can analyse a list, so that if L represents the empty list then

$$\mathit{Listcases}(P, (v\ell')Q)\langle\ell\rangle \mid L\langle\ell\rangle \rightarrow^* P$$

while if L represents a list whose head and tail are represented by V and L' then

$$\mathit{Listcases}(P, (v\ell')Q)\langle\ell\rangle \mid L\langle\ell\rangle \rightarrow^* \mathsf{new}\, v\ell'\, (Q \mid V\langle v\rangle \mid L\langle\ell'\rangle)$$

Demonstrate this last reaction. [6 marks]

Write down a sorting respected by the list abstractions and *Listcases*, involving a parametric sort $\textsc{list}(\sigma)$ (where σ may be any sort for the elements of a list). [3 marks]