1999 Paper 9 Question 14

Numerical Analysis II

With reference to solution of the differential equation $y' = f(x, y)$, explain the conventional notation x_n, $y(x_n)$, y_n, and f_n. [3 marks]

Explain the terms *local error*, *global error*, and *order* of a method. [3 marks]

Milne's method uses the *multistep* formulae

$$y_{n+1} = y_{n-3} + \frac{4h}{3}(2f_n - f_{n-1} + 2f_{n-2})$$

$$y_{n+1} = y_{n-1} + \frac{h}{3}(\tilde{f}_{n+1} + 4f_n + f_{n-1})$$

which each have local error $O(h^5)$. Outline the general technique for deriving multistep formulae. What is the meaning of the term $\tilde{f}_{n+1}$? Suggest a suitable starting procedure and explain how the Milne formulae are used. [8 marks]

Let $y(x_0) = 0, h = 0.3$ and $f(x, y) = 3y/x - 2$. Suppose the following values of f_n have been generated by the starting procedure: 1.3, 2.1, 3.4 for $n = 1, 2, 3$. Calculate the first required value of $\tilde{f}_{n+1}$. [3 marks]

Contrast Milne's method with a comparable *one step* method, commenting particularly on *stability*, *efficiency* and *step size* considerations. [3 marks]