1999 Paper 7 Question 9

Denotational Semantics

Suppose that $f : D \to D$ is a continuous function on a domain. What is meant by the *least pre-fixed point*, $\mathit{fix}(f)$, of f? [2 marks]

Show that $\mathit{fix}(f)$ exists and is in fact the least fixed point of f. [12 marks]

Suppose now that E is another domain and $g : D \times E \to E$ a continuous function. Let (d, e) be the least element of $D \times E$ satisfying

$$\begin{cases} d &= f(d) \\ e &= g(d, e) \end{cases}$$

Prove that $d = \mathit{fix}(f)$. [6 marks]