## 1999 Paper 2 Question 7

## Regular Languages and Finite Automata

Suppose that $L$ is a language over the alphabet $\{0,1\}$. Let $L^{\prime}$ consist of all strings $u^{\prime}$ over $\{0,1\}$ with the property that there is some string $u \in L$ with the same length as $u^{\prime}$ and differing from $u^{\prime}$ in at most one position in the string. Show that if $L$ is regular, then so is $L^{\prime}$. [Hint: if $Q$ is the set of states of some finite automaton accepting $L$, construct a non-deterministic automaton accepting $L^{\prime}$ with states $Q \times\{0,1\}$, where the second component counts how many differences have been seen so far.]
[10 marks]

If a deterministic finite automaton $M$ accepts any string at all, it accepts one whose length is less than the number of states in $M$. Explain why.

State Kleene's theorem about regular expressions and deterministic finite automata.
[2 marks]
Describe how to decide for any given regular expression whether or not there is a string that matches it.

