1999 Paper 2 Question 5

Probability

A solitaire game is played with two coins. One coin is fair, so that $P(\text{heads}) = \frac{1}{2}$, and the other is biased so that $P(\text{heads}) = p$.

At the first turn the player tosses the fair coin. At all subsequent turns the biased coin is used if the toss at the previous turn resulted in heads and the fair coin is used if the toss at the previous turn resulted in tails.

Suppose u_n is the probability of obtaining heads at turn n. Show that, for $n > 1$:

$$2u_n + (1 - 2p)u_{n-1} = 1 \qquad \text{[4 marks]}$$

Demonstrate that this difference equation also holds for the case $n = 1$ provided u_0 is suitably defined. [2 marks]

Solve the difference equation, expressing u_n in terms of n and p. [14 marks]