COMPUTER SCIENCE TRIPOS Part IB

Tuesday 1 June 1999 1.30 to 4.30

Paper 4

Answer **five** *questions.*

Submit the answers in five **separate** *bundles, each with its own cover sheet. On each cover sheet, write the numbers of* **all** *attempted questions, and circle the number of the question attached.*

Write on **one** *side of the paper only.*

1 Computation Theory

Define the *primitive recursive* and *partial* ($\mu-$) *recursive* functions. [6 marks]

Suppose you are given a Turing machine with state set Q and k-symbol alphabet S whose action is defined by transition functions

$$\begin{aligned} q' &= f(q,s) \in Q \uplus \{H\} && \text{(disjoint union)} \\ s' &= r(q,s) \in S && \text{(replacement symbol)} \\ d' &= d(q,s) \in \{L,R,C\} && \text{(movement)} \end{aligned}$$

where the head moves to L or R on the tape unless $q' = H$, in which case $d' = C$ and the machine stops.

Extend the action of the machine by an additional state symbol D so that for all $s \in S$,

$$\begin{aligned} f(H,s) &= f(D,s) = D \\ r(H,s) &= r(D,s) = s \\ d(H,s) &= d(D,s) = C \end{aligned}$$

Show that the action of the Turing machine as extended in this way can be described by a primitive recursive function $T(t,x)$, where t is a step counter and x is a code specifying the initial configuration. [10 marks]

Hence show that computation by any Turing machine may be represented by a partial recursive function. [4 marks]

2 Concurrent Systems

Assume that on a power fail the entire contents of main memory are lost. Discuss the various problems that can be caused and how they might be solved in

either a centralised transaction processing system

or a remote procedure call protocol

[20 marks]

3 Further Java

Describe the facilities in Java for restricting concurrent access to critical regions. Explain how shared data can be protected through the use of objects. [8 marks]

The built-in facilities for restricting concurrency in Java allow only one thread at a time to be executing within the critical region. A different approach is to distinguish *shared* and *exclusive* access to a critical region: any number of *readers* may share access at the same time, but only one *writer* may acquire exclusive access (excluding any readers while it does so).

Specify a `MultiSync` class in Java with methods to acquire and release both read and write access, and sketch its implementation. [6 marks]

Derive a `MultiBuffer` class that extends `MultiSync` with methods to store and read a data field, ensuring that any locks are released when a waiting thread is interrupted. Your example may use a simple data field such as an integer but you should explain why such an elaborate scheme of concurrency control is unnecessary in this case. [6 marks]

4 Compiler Construction

It is commonly suggested that Algol-60 call-by-name can be modelled by passing a function as a call-by-value parameter. Show how a program containing a definition

```
int f(int x:name) { ... x ... x ... }
```

of `f` (where `x` occurs only in Rvalue context) and a call `f(`e`)` to `f` can be replaced by an equivalent definition and call using only call-by-value. [6 marks]

Most such explanations assume that the uses of `x` within `f` occur only in Rvalue context. However, Algol-60 also permits the equivalent of

```
int g(int x:name) { if (p) { ... x := x+1; x := -x; ... }
                        return x;
                  }
```

and calls like `g(a[k()])` which, when `p` is `true`, would have the effect of calling `k()` five times and consequent access to five (possibly different) subscripts of array `a[]`. Develop your explanation for the first part of this question to cover also the case of a call-by-name parameter being used in both Lvalue and Rvalue contexts. [Hint: note that when `p` is `false` then the actual parameter to `g` need not be an Lvalue, so you may need two parameterless procedure arguments ("thunks").]

[8 marks]

Using the previous part or otherwise, give a translation of a definition and call `h(`e`)` using call-by-value-result (Ada `in out` mode) with no uses of the address-of (`&`) operator other than those involved in call-by-name. Your explanation is allowed to deviate from call-by-value-result by allowing side-effects in e to take place twice.

[6 marks]

5 Data Structures and Algorithms

Outline the mechanism used in the Burrows–Wheeler block compression algorithm, illustrating your description by applying it to the string ALFALFA. [14 marks]

Briefly discuss the advantages and disadvantages of the Burrows–Wheeler algorithm compared with other commonly used compression methods. [6 marks]

6 Computer Design

A modern processor's memory is constructed from a hierarchy of memories. Explain what the levels of the hierarchy are. [5 marks]

Why do modern processors have several registers rather than an accumulator plus one or two index registers? [5 marks]

Hand compile the following pseudo-code for both a register machine and an accumulator machine of your choice. You may invent an instruction set. Make the semantics of the code clear via comments.

```
a=0;
b=1;
for(i=0; i<5; i++) {
    a=a+b;
    b=a-b;
}
```

[10 marks]

7 Operating System Functions

The following are three ways which a file system may use to determine which disk blocks make up a given file.

(*a*) chaining in a map

(*b*) tables of pointers

(*c*) extent lists

Briefly describe how each scheme works. [3 marks each]

Describe the benefits and drawbacks of using scheme (*c*). [6 marks]

You are part of a team designing a distributed filing system which replicates files for performance and fault-tolerance reasons. It is required that rights to a given file can be revoked within T milliseconds ($T \geqslant 0$). Describe how you would achieve this, commenting on how the value of T would influence your decision. [5 marks]

8 Continuous Mathematics

Suppose we need to solve a linear second-order differential equation with constant coefficients A and B in which the combined derivatives of the solution we seek, $f(x)$, are only known to be related to another function, $g(x)$:

$$A\frac{d^2f(x)}{dx^2} + B\frac{df(x)}{dx} = g(x)$$

We know the function $g(x)$, and we can compute its Fourier Transform $G(\mu)$.

How can we use the properties of the Fourier Transform and its inverse in order to compute the solution $f(x)$ of this differential equation? Provide an expression for $F(\mu)$, the Fourier Transform of $f(x)$, in terms of $G(\mu)$, frequency variable μ, and the coefficients in the differential equation. [8 marks]

What final step is now required in order to compute the actual solution $f(x)$ of the differential equation, given your expression for $F(\mu)$? [2 marks]

In numerical computing, differential operators must always be represented by finite differences. Assume that a function has been sampled at uniform, closely spaced, intervals. How many consecutive sample points are necessary in order to compute the N^{th} derivative of the function at some point? [2 marks]

To compute the third derivative in a local region of a function, what set of weights would you use to multiply consecutive samples of the function? [3 marks]

What is the principal computational advantage of using orthogonal functions, over non-orthogonal ones, when representing a set of data as a linear combination of a universal set of basis functions? [2 marks]

If $\Psi_k(x)$ belongs to a set of orthonormal basis functions, and $f(x)$ is a function or a set of data that we wish to represent in terms of these basis functions, what is the basic computational operation we need to perform involving $\Psi_k(x)$ and $f(x)$? [3 marks]

9 Numerical Analysis I

The mid-point rule can be expressed in the form

$$I_n = \int_{n-\frac{1}{2}}^{n+\frac{1}{2}} f(x)dx = f(n) + e_n$$

where

$$e_n = f''(\theta_n)/24$$

for some θ_n in the interval $(n - \frac{1}{2}, n + \frac{1}{2})$. Assuming that a formula for $\int f(x)dx$ is known, and using the notation

$$S_{p,q} = \sum_{n=p}^{q} f(n),$$

describe a method for estimating the sum of a slowly convergent series $S_{1,\infty}$, by summing only the first N terms and estimating the remainder by integration.

[6 marks]

Assuming that $f''(x)$ is a positive decreasing function, derive an estimate of the error $|E_N|$ in the method. [5 marks]

Given

$$\int \frac{dx}{x(x+2)} = -\frac{1}{2}\log_e(1 + \frac{2}{x})$$

illustrate the method by applying it to

$$\sum_{n=1}^{\infty} \frac{1}{n(n+2)}.$$

Verify that $f''(x)$ is positive decreasing for large x, and estimate the integral remainder to be added to $S_{1,N}$. [You may assume $\log_e(1 + \lambda) \simeq \lambda$ for λ small.]

[6 marks]

To 2 significant digits, how large should N be to achieve an absolute error of approximately 1.8×10^{-11}? [3 marks]

10 Computer Graphics and Image Processing

Give the formula for a Bezier cubic curve.

Derive the conditions necessary for two Bezier cubic curves to join with (*a*) just $C0$-continuity and (*b*) $C1$-continuity.

Give a geometric interpretation of each condition in terms of the locations of control points.

Explain (mathematically) why a Bezier cubic curve is guaranteed to lie within the convex hull of its control points. [8 marks]

Basic ray tracing uses a single sample per pixel. Describe *four* distinct reasons why one might use multiple samples per pixel. Explain the effect that each is trying to achieve, and outline the mechanism by which it achieves the effect. [8 marks]

Describe the differences in the computational complexity of the depth sort and binary space partition (BSP) tree algorithms for polygon scan conversion.

If you were forced to choose between the two algorithms for a particular application, what factors would be important in your choice? [4 marks]