## 1998 Paper 9 Question 7

## **Optimising Compilers**

Consider the programming language with terms e having abstract syntax:

$$e ::= x | c | \lambda x.e | e_1 e_2 | let x = e_1 in e_2$$

where x ranges over a set of identifiers and c over a set of integer constants. For the rest of the question, your answers can be illustrated by reference to the program p:

$$\lambda z.let \ id = \lambda x.x \ in \ id \ id \ 7$$

State how to label terms in p uniquely so that a subterm occurring repeatedly in a term has different labels. [4 marks]

Show how such terms may be seen as a family of flowgraphs, one for each  $\lambda$  (you may find it useful to consider the above labelling as providing a unique function name for anonymous  $\lambda$ -abstractions). [4 marks]

Define the *call graph* of such a family of flowgraphs, stating clearly how indirect calls are treated. [4 marks]

Describe how to associate a flow-variable with each labelled node of a term such as p and to derive equations which can improve the above treatment of indirect calls to get a better approximation of the edges in the call graph. [8 marks]

[Hint: you may find it useful to recall the shorthand of  $(\gamma \mapsto \delta) \supseteq \beta$  as representing the compound constraint that

whenever 
$$(\lambda x^j . e^k)^i \in \beta$$
 we have  $\alpha_j \supseteq \gamma \land \delta \supseteq \alpha_k$ 

where  $\alpha_r$  is the flow variable associated with the node labelled r.]