1998 Paper 9 Question 11

Information Theory and Coding

Consider Shannon's third theorem, the *Channel Capacity Theorem*, for a continuous communication channel having bandwidth W Hertz, perturbed by additive white Gaussian noise of power spectral density N_0, and average transmitted power P.

(a) Is there any limit to the capacity of such a channel if its signal-to-noise ratio $\frac{P}{N_0 W}$ can be arbitrarily increased? If so, what is that limit? [2 marks]

(b) Is there any limit to the capacity of such a channel if, leaving N_0 and P unchanged, its bandwidth W in Hertz can be arbitrarily increased? If so, what is that limit? [3 marks]

[Hint: $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$ where $-1 < x < 1$.]

Explain why smoothing a signal, by low-pass filtering it *before* sampling it, can prevent aliasing. Draw a graph in the Fourier domain which illustrates the origin of aliasing, and also show in the picture how smoothing solves the problem. What would be the most effective low-pass filter to use for this purpose? Draw it in the Fourier domain. [10 marks]

Suppose that women who live beyond the age of 70 outnumber men in the same age bracket by three to one. How much information, in bits, is gained by learning that a certain person who lives beyond 70 happens to be male? [5 marks]