# 1998 Paper 8 Question 14

**Numerical Analysis II**

Let $n_+$ be the number of positive real roots of a polynomial $p_n(x)$. Let $c$ be the number of changes of sign when the coefficients are taken in order. State *Descartes' rule of signs*. [2 marks]

If
$$p_3(x) = x^3 - 8x^2 + 11x + 20$$

what does this rule say about the polynomials $p_3(x)$, $p_3(-x)$? [2 marks]

Newton's method for solution of a system of $n$ non-linear equations $\mathbf{f}(\mathbf{x}) = \mathbf{0}$ can be expressed in the form
$$\mathbf{x}_{k+1} = \mathbf{x}_k + \mathbf{h}_{k+1}.$$

What is the formula for $\mathbf{h}_{k+1}$? [2 marks]

Outline the *damped Newton method* in $n$ variables. [4 marks]

Apply the damped Newton method in one variable to find one root of the polynomial $p_3(x)$, using $x_0 = 1$ as the starting value. Hence factorise $p_3(x)$ and draw a rough sketch of the polynomial, showing the first Newton step as a tangent. [10 marks]