

# 1998 Paper 4 Question 1

## Continuous Mathematics

Consider the family of one-dimensional Gabor wavelets, parameterised for location  $x_0$ , size  $\alpha$ , and frequency  $\mu_0$ . Their functional form is:

$$f(x) = e^{-i\mu_0 x} e^{-(x-x_0)^2/\alpha^2}$$

and their Fourier Transform is:

$$F(\mu) = e^{-ix_0\mu} e^{-(\mu-\mu_0)^2\alpha^2}$$

- (a) Explain the dualities of: (i) modulation and shifting; and (ii) similarity (reciprocal scaling), in terms of the behaviour of the parameters in the expressions above for  $f(x)$  and  $F(\mu)$ . [4 marks]
- (b) What can you say about the Fourier transform of the *sum* of any two Gabor wavelets? [4 marks]
- (c) What can you say about the Fourier transform of the *product* of any two Gabor wavelets, and why? [4 marks]
- (d) What is the Fourier transform of  $f^{(n)}(x)$ , the  $n^{\text{th}}$ -derivative of a Gabor wavelet? [4 marks]
- (e) Show that the set of all Gabor wavelets is closed under convolution: *i.e.*, that the convolution of any two Gabor wavelets is itself a single Gabor wavelet. [4 marks]