

1998 Paper 12 Question 12

Complexity Theory

- (a) Describe the problem 3-SAT. [2 marks]
- (b) Show how any instance of the seemingly more general problem n -SAT can be reduced to an equivalent one where each term has *exactly* three literals in it. Estimate how much larger the reduced problem would be than the original one. [4 marks]
- (c) A certain computation using a non-deterministic Turing machine completes in T time-steps. The Turing machine has k states and uses an alphabet of N symbols. A major theorem underpinning the concept of NP-completeness is based on a conversion of a description of such computations to boolean formulae which characterise them.

Explain how, in such a reduction, boolean variables may be used to describe states that the Turing machine might be in. Show how to derive those components of the boolean formula that relate just to the way in which the Turing machine moves its read–write head. Your explanation should be sufficiently complete and carefully explained that it could be used as a specification of a program that would perform that part of the translation from Turing machine descriptions to boolean formulae. You should not attempt to explain the rest of the boolean formula or how it fits into a complete proof or program. [11 marks]

- (d) In terms of T , k and N , about how many symbols does it take to write the boolean expression you generate? [3 marks]