

## 1998 Paper 11 Question 10

### Numerical Analysis I

What are the three basic operations used in *Gaussian Elimination with partial pivoting*? [3 marks]

Consider the equations

$$\begin{pmatrix} 5 & 5 & 9 \\ 1 & 0.99 & 100 \\ 1 & 2 & 3.8 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0.5 \\ 100 \\ 2.1 \end{pmatrix}$$

Perform only the operations described below. Be careful to ensure that results and all intermediate values are rounded to **only 2 significant decimal digits**. [A calculator may be used, but is not essential.]

- (a) Using the first equation as pivot, obtain two equations in  $x_2$  and  $x_3$ . [4 marks]
- (b) Solve the remaining two equations *without* interchanging equations. Obtain a value for  $x_3$ . [2 marks]
- (c) Solve the same two equations again *with* interchange of equations. Show that the same value of  $x_3$  is obtained to 2 significant digits. [2 marks]
- (d) Use the method of *back substitution* twice to obtain a pair of solutions  $\{x_1, x_2, x_3\}$  corresponding to steps (b) and (c). [4 marks]
- (e) By substituting your results into the original equations, compute vectors of residual errors. Using any suitable norm, determine which of the pair of solutions is more accurate. [5 marks]