1997 Paper 9 Question 14

Numerical Analysis II

Explain the term *positive semi-definite*. [1 mark]

Let $\mathbf{A}$ be a square matrix. State *Schwarz's inequality* for the product $\mathbf{Ax}$. What are the *singular values* of $\mathbf{A}$, and how are they related to the ℓ_2 norm of $\mathbf{A}$? [4 marks]

Describe briefly the *singular value decomposition* of the matrix $\mathbf{A}$, and how it may be used to solve the linear equations $\mathbf{Ax} = \mathbf{b}$. [4 marks]

Let $\hat{\mathbf{x}}$ be an approximate solution of $\mathbf{Ax} = \mathbf{b}$, and write $\mathbf{r} = \mathbf{b} - \mathbf{A}\hat{\mathbf{x}}$, $\mathbf{e} = \mathbf{x} - \hat{\mathbf{x}}$. Find an expression for the relative error $\|\mathbf{e}\|/\|\mathbf{x}\|$ in terms of computable quantities. Show how your formula is related to the *singular values* of $\mathbf{A}$. [8 marks]

How may this formula be used if some *singular values* are very small? [3 marks]