1997 Paper 5 Question 9

Complexity Theory

Comment on each of the following assertions, explaining whether they are right, wrong or imprecisely stated. State explicitly any standard results needed to justify your assertions and, in cases where the statement made is almost but not quite correct, attempt to clarify or mend it.

- (a) If one had a special piece of hardware that could solve the boolean-satisfiability problem 3-SAT in time $n \log(n)$ then there would be some constant K such that it would be possible to solve all NP-complete problems in time no worse than n^{K} .
- (b) The square root of an *n*-bit number can be computed in time proportional to $n \log(n) \log \log(n)$ on an ordinary computer, therefore computing square roots is not an NP problem.
- (c) Every task that can be performed in Polynomial Time will be solvable on a conventional computer in a reasonable amount of time.
- (d) Any task that is NP will take an unreasonable amount of time if an attempt is made to solve it using an ordinary computer.
- (e) Deciding whether black or white will win the game of chess if both players behave totally logically is not an NP problem because it is of finite size.

[20 marks]