## 1997 Paper 2 Question 1

Twenty-part question (One mark per part)
(a) Give some ML text to replace <insert> in the following:

```
<insert>
fun f g g = g; f x y;
```

to make it into a valid ML program.
(b) Given the following ML function definition:
fun h x y z = x (y z) ;
what is the type of h ?
(c) Let $A$ be the set $\{1,2,3,4\}$. Exhibit (or state briefly why this is impossible) a relation $R$ on $A$ which is a function whose inverse relation $R^{-1}$ is a partial function, but not a (total) function.
(d) Solve the recurrence

$$
\begin{aligned}
f(0) & =0 \\
f(1) & =3 \\
f(n+2) & =6 f(n+1)-9 f(n)
\end{aligned}
$$

(e) It is claimed that there is a bijection from $(A+B) \rightarrow C$ to $(A \rightarrow C) \times(B \rightarrow C)$. Demonstrate this by replacing both "..." below with valid ML code which might form such a bijection and its inverse. Just ML is required, not proof.

```
datatype BplusS = left of bool | right of string;
fun LtoR(f: BplusS -> real)
    : (bool->real) * (string->real)
    = ...;
fun RtoL((g: bool->real), (h: string->real))
    : (BplusS -> real)
    = ...;
```

( $f$ ) Give an equation involving a set named $S$, such that the smallest (with respect to $\subseteq$ ) solution for $S$ is the set of values represented by the ML datatype s:

```
datatype s = empty | leaf of int | branch of s*s;
```

Your solution may reasonably involve the constructors empty, leaf and branch.
(g) Given an integer value between 0 and 9 , show an expression in Modula-3 for the equivalent character.
( $h$ ) What output would the following Modula-3 fragment produce when P was called?

```
PROCEDURE P () =
    PROCEDURE Q (VAR a: INTEGER) =
            BEGIN a := 2 * a + b END Q;
        VAR a := 1; b := 2; c := 3;
        BEGIN
            Q (c);
            IO.Put (Fmt.Int (a) & "\n");
            IO.Put (Fmt.Int (b) & "\n");
            IO.Put (Fmt.Int (c) & "\n");
        END P;
```

(i) What is meant by an address space?
(j) How are operating system services invoked?
(k) What is this?

( $l$ ) What is the minimum sum of product form for the function specified by the following map in which X indicates don't care?

( $m$ ) Identify at least one other factor involved in computer security beyond technical measures such as provision of password access.
( $n$ ) Name two branches of the law under which software ownership can be protected.
(o) Provide a generating function $G(\eta)$ that is suitable for a random variable which is distributed $\operatorname{Geometric}(p)$.
(p) Given that $\mathrm{P}(X=0)=\mathrm{P}(Y=0)=\frac{1}{4}$ and that $\mathrm{P}(X=0 \cap Y=0)=\frac{1}{8}$, determine $\mathrm{P}(X \neq 0 \cap Y \neq 0)$.
(q) Describe the way in which Regular Expressions are constructed.
( $r$ ) How does the minimum time in which a large software project can be completed depend on the number of staff allocated to it?
(s) Give in binary a pair of five-bit numbers which when added give the correct result when considered as unsigned integers but which overflow when considered as two's complement numbers.
(t) What is meant by top-down programming and how does it differ from bottom-up programming?

