1997 Paper 1 Question 6

Foundations of Computer Science

A rooted directed graph has vertices identified by integers. Each vertex `v` has a left successor given by `left(v)` and a right successor given by `right(v)`, where `left` and `right` are ML functions of type `int->int`. The graph contains the root and all vertices reachable by paths from the root.

Define a datatype `G` that could be used to represent such a graph with given root, and left and right functions, and define a function `mkgraph(root, left, right)` that can create values of type `G`. Show that such values can be used to represent both finite and infinite graphs. [4 marks]

A path through the graph is represented by a `bool list` with `true` and `false` indicating left and right edges, respectively.

Define the function `last : G -> bool list -> int` that will yield, for a given graph, the identity of the vertex reached by following the given path from the root. [3 marks]

In a new application, where `last` is repeatedly called, it is required for it to return both the identity of the last vertex and a count of how often this particular vertex has been returned. Define a new version of the datatype `G`, containing mutable values, that could be used. [3 marks]

Illustrate the use of this datatype by defining the new versions of `mkgraph` and `last`. [10 marks]