

1997 Paper 1 Question 5

Foundations of Computer Science

Noughts and Crosses is a game played by two players (O and X) on a board with nine positions numbered as follows:

1	2	3
4	5	6
7	8	9

The players place their marks (O and X) in unoccupied positions on the board until the game is complete. A completed game is when either

- (a) there is a straight line of three Xs giving a win for X, *or*
- (b) there is a straight line of three Os giving a win for O, *or*
- (c) all nine positions are occupied, in which case the game is drawn.

O is the first player to move.

It is required to construct an ML structure representing the tree of all possible games. Each node of the tree should represent a reachable board state, with the root being the empty board, and the leaf nodes corresponding to won, lost or drawn games.

Define the ML datatype `tree` that you would use to represent this game tree.

[3 marks]

Define the function `mktree : unit->tree` to construct the complete game tree, explaining carefully how it works. There is no need for your implementation to be efficient in either space or time.

[10 marks]

Briefly discuss ways in which your implementation of `mktree` could be made more efficient.

[4 marks]

Define a function `Owins : tree->int` which when applied to the complete tree will yield the number of distinct games in which O wins.

[3 marks]