1996 Paper 8 Question 3

Computer System Modelling

A telephone exchange multiplexes 64 Kb/s voice calls onto a 256 Kb/s trunk line (therefore the line will hold at most four calls). New calls have an exponentially distributed inter-arrival process, with a mean of 20 seconds, and the call holding time is exponentially distributed with a mean of 60 seconds.

- (a) Draw a diagram of a Markov Chain which models the system, labelling the state transitions with their rates where appropriate. What is the necessary condition for stability of this system?
- (b) Derive an expression for the probability that an arriving call finds k calls in progress, for $k \ge 0$, and thence calculate the probability that a caller finds the exchange engaged, given the parameters above. [15 marks]