

1996 Paper 8 Question 15

Pi Calculus

What are the three kinds of *commitment* for a process in the π calculus? Explain informally how the commitments of $P \mid Q$ arise from those of P or Q or both.

[6 marks]

We wish to implement a queue, to be weakly equivalent to the specification



$$Q\langle \rangle \stackrel{\text{def}}{=} \text{join}(x).Q\langle x \rangle$$

$$Q\langle x_1 \dots x_n \rangle \stackrel{\text{def}}{=} \text{join}(x).Q\langle x_1 \dots x_n x \rangle + \overline{\text{serve}}\langle x_1 \rangle.Q\langle x_2 \dots x_n \rangle$$

The body of the queue is to consist of a chain of cells, each having the form $\bar{a}\langle x, b \rangle$; to hold a queue of n items $x_1 \dots x_n$ we define a chain parametrically on its head a_0 and tail a_n as follows:

$$\text{Cells}\langle x_1 \dots x_n \rangle \stackrel{\text{def}}{=} (a_0 a_n)(\nu a_1 \dots a_{n-1})(\bar{a}_0\langle x_1, a_1 \rangle \mid \dots \mid \bar{a}_{n-1}\langle x_n, a_n \rangle)$$

- (a) Define agents $\text{Server}\langle a \rangle$ and $\text{Joiner}\langle b \rangle$ to manage the head and tail of the queue, respectively, with the intention that the system $\text{Queue}\langle x_1 \dots x_n \rangle$ defined below should be weakly equivalent to $Q\langle x_1 \dots x_n \rangle$:

$$\text{Queue}\langle \rangle \stackrel{\text{def}}{=} (\nu a)(\text{Server}\langle a \rangle \mid \text{Joiner}\langle a \rangle)$$

$$\text{Queue}\langle x_1 \dots x_n \rangle \stackrel{\text{def}}{=} (\nu ab)(\text{Server}\langle a \rangle \mid \text{Cells}\langle x_1 \dots x_n \rangle\langle ab \rangle \mid \text{Joiner}\langle b \rangle)$$

[7 marks]

- (b) In terms of your definition of Server and Joiner , work out the commitments of $\text{Queue}\langle x_1 \dots x_n \rangle$ far enough to give an informal argument that it is indeed weakly equivalent to $Q\langle x_1 \dots x_n \rangle$.

[Hint: treat the cases $n = 0$ and $n \neq 0$ separately.]

[7 marks]