1996 Paper 5 Question 10

Logic and Proof

For each of the following formulæ, construct a proof in the tableau calculus or show that no proof exists.

$$\begin{array}{ll} ((A \to B) \to A) \to A & [4 \text{ marks}] \\ \forall z \, \exists x \, \forall y \, ((P(y) \to Q(z)) \to (P(x) \to Q(x))) & [12 \text{ marks}] \\ \Box (A \to B) \land \Box (B \to A) & [4 \text{ marks}] \end{array}$$

Assume S4 modal logic for the last one.