1996 Paper 11 Question 9

Computation Theory

A *bag* B of natural numbers is a total function $f_B : \mathbb{N} \to \mathbb{N}$ giving for each natural number x the count $f_B(x)$ of occurrences of x in B. If each $f_B(x) = 0$ or 1, then f_B is the characteristic function χ_s of a set S: every set can thus be regarded as a bag.

(a) A bag B is *recursive* if the function f_B is computable. Suppose that the sequence of bags $\{B_n \mid n \in \mathbb{N}\}$ is recursively enumerated by the computable function $e(n, x) = f_n(x)$, which gives the count of x in each bag B_n. Show that there is a recursive set S that is different from each bag B_n. [7 marks]

Hence prove that the set of all recursive bags cannot be recursively enumerated. [3 marks]

(b) A bag B is *finite* if there is $X \in \mathbb{N}$ such that $f_B(x) = 0$ for all $x \geqslant X$. Show that the set of all finite bags is recursively enumerable. [10 marks]