1995 Paper 9 Question 15

Numerical Analysis II

With reference to solution of the differential equation $y' = f(x, y)$, explain the conventional notation x_n, $y(x_n)$, y_n, f_n. [4 marks]

Derive Euler's method

$$y_{n+1} = y_n + hf(x_n, y_n). \tag{1}$$

[3 marks]

Euler's method has *local error*

$$\frac{h^2}{2} y''(\xi).$$

Explain the terms *local error*, *global error*. [2 marks]

The multistep formula

$$y_{n+1} = y_{n-3} + \frac{4h}{3}(2f_n - f_{n-1} + 2f_{n-2}) \tag{2}$$

has *local error*

$$\frac{14}{45} h^5 y^{(5)}(\xi).$$

Outline the technique for deriving multistep formulae such as (2). (Omit algebraic details.) [2 marks]

Suppose Euler's formula is used as a starting procedure for formula (2). How many initial steps of formula (2) need to be evaluated using Euler? [2 marks]

Estimate very roughly the number N of Euler steps needed to approximate f_1. (Assume that $\mid y^{(5)}(x) \mid \simeq 30$, and Euler's method has *global error* h/N.) [5 marks]

What is the most important requirement for a starting procedure? Suggest a more suitable starting procedure than Euler. [2 marks]