1995 Paper 6 Question 9

Logic and Proof

Briefly describe propositional logic, first-order logic and modal logic. Outline the syntax and semantics of each. [5 marks]

Describe the main features of the sequent calculus. (It is not necessary to give the rules for particular connectives.) [2 marks]

Explain informally why the following sequent calculus rules are sound, adding provisos if necessary:

$$\frac{A, B, \Gamma \Rightarrow \Delta}{A \land B, \Gamma \Rightarrow \Delta} \qquad \qquad \frac{A, \Gamma \Rightarrow \Delta}{\exists x A, \Gamma \Rightarrow \Delta} \qquad \qquad [4 \text{ marks}]$$

Prove $\exists x(A \land B) \Rightarrow (\exists xA) \land B$ using the sequent calculus. Add any necessary provisos and explain why they are necessary. [9 marks]