

1995 Paper 4 Question 10

Numerical Analysis I

The Newton–Raphson formula for solution of $f(x) = 0$ is

$$\hat{x} = x - \frac{f(x)}{f'(x)}.$$

By means of a sketch graph, describe how the method works in a simple case.

[4 marks]

When the method converges, what *rate of convergence* is expected? Describe one circumstance in which the method may fail to converge.

[4 marks]

Consider the simultaneous equations

$$\left. \begin{aligned} f_1(x_1, x_2) &= x_2 - x_1^2 - 2 = 0 \\ f_2(x_1, x_2) &= x_1(x_2 - 3x_1) = 0 \end{aligned} \right\} \quad (1)$$

Suppose the iterative scheme

$$\begin{pmatrix} -2x_1 & 1 \\ x_2 - 6x_1 & x_1 \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} -f_1(x_1, x_2) \\ -f_2(x_1, x_2) \end{pmatrix} \quad (2)$$

is applied to the equations (1). If $\{x_1, x_2\}$ is the starting approximation, the improved approximation is given by

$$\begin{pmatrix} \hat{x}_1 \\ \hat{x}_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}.$$

Suppose $x_1 = 0$. Show, by solving the equations (2) that the first iteration always produces the same improved approximation for any non-zero x_2 .

[10 marks]

Verify that the method converges if x_1 is set to 0, and $x_2 \neq 0$.

[2 marks]