# 1995 Paper 3 Question 9

**Computation Theory**

Give a brief explanation of each of the following:

(*a*)  the encoding of register machine programs as numbers  [5 marks]

(*b*)  the notion of a *universal* register machine  [3 marks]

(*c*)  the notion of a register machine *decidable* subset of $\mathbb{N} = \{0, 1, 2, \ldots\}$  [2 marks]

Assuming the existence of a universal register machine, or otherwise, show that there is a unary computable partial function $f(x)$ such that both $\{x \in \mathbb{N} \mid f(x) \text{ is defined}\}$ and $\{y \mid \text{for some } x,\ f(x) = y\}$ are register machine undecidable. If you appeal to the undecidability of the Halting Problem, you must prove it. (You may assume the existence of register machines for coding and decoding pairs of numbers as numbers.)  [10 marks]

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