1994 Paper 9 Question 12

Semantics of Programming Languages

State the Tarski-Knaster fixed-point theorem. Give a brief justification for the importance of the fixed-point theorem in denotational semantics. [8 marks]

Prove that the least fixed-point operator fix is a continuous function. You may assume the following result:

> Let $\langle D, \sqsubseteq \rangle$ be a complete partial order. Every doubly-increasing chain $\langle d_{ij} \rangle_{i,j \in \omega}$ in D (i.e. for any i, j, i', j' in ω, if $i \leqslant i'$ and $j \leqslant j'$ then $d_{ij} \sqsubseteq d_{i'j'}$) has a least upper bound l. Further,
>
> $$l \quad = \quad \bigsqcup_{i \in \omega} \bigsqcup_{j \in \omega} d_{ij} \quad = \quad \bigsqcup_{j \in \omega} \bigsqcup_{i \in \omega} d_{ij} \quad = \quad \bigsqcup_{k \in \omega} d_{kk}$$

[12 marks]