1994 Paper 9 Question 10

Numerical Analysis II

Apply the Newton-Raphson formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

for solving the equation $f(x) = 0$ using $f(x) = x - \frac{1}{2}e^{1-x^2}$. Simplify your formula so that the exponential function need be called only once per iteration. [4 marks]

This function has a zero at approximately 0.76. Use $x_0 = 1$ and perform one Newton-Raphson iteration to calculate x_1 and verify that this is a good starting value. Now use $x_0 = -1/\sqrt{2}$ and re-calculate x_1. (You may assume for this purpose: $\sqrt{2} \simeq 1.4$, $1/\sqrt{2} \simeq 0.71$, $\sqrt{e} \simeq 1.6$, $1/\sqrt{e} \simeq 0.61$.) [3 marks]

By sketching the graph of $f(x)$, or otherwise, explain these results. [4 marks]

Perform two Newton-Raphson iterations (in exact arithmetic) for the function $f(x) = x - 1/(x+1)$, with $x_0 = 1$. Suppose that the second iteration is modified as follows:

$$\tilde{x}_2 = x_1 - \frac{f(x_1)}{f'(x_0)}.$$

Examine $|x_2 - \tilde{x}_2|/|x_2|$ to estimate the relative loss of accuracy if the modified method is used. [5 marks]

Although $\tilde{x}_2$ is obviously less accurate than x_2 in general, explain briefly why this modified Newton method is more useful for an n-dimensional problem. [4 marks]