1994 Paper 7 Question 10

Numerical Analysis II

Explain the terms *Riemann integral* and *Riemann sum.* [3 marks]

Let $\mathbf{R}$ be a quadrature rule that integrates constants exactly. If a function f is bounded and Riemann-integrable over the interval $[a, b]$ then prove that

$$\lim_{n\to\infty} (n \times \mathbf{R})f = \int_a^b f(x)dx.$$ [6 marks]

Consider two quadrature rules for the interval $[-\lambda, \lambda]$:

$$\begin{aligned}\mathbf{S}f &= \frac{\lambda}{3}\{f(-\lambda) + 4f(0) + f(\lambda)\} - \frac{\lambda^5}{90}f^{(4)}(\xi)\\ \mathbf{T}f &= \lambda\{f(-\lambda) + f(\lambda)\} - \frac{2}{3}\lambda^3 f''(\zeta)\end{aligned}$$

If $\mathbf{S}$ were used in the composite form $(n \times \mathbf{S})f$, what order of convergence would you expect? [2 marks]

Suppose the rule

$$\begin{aligned}&\frac{1}{3}\{F(-1,-1) + 4F(-1,0) + F(-1,1)\\ &\quad + F(1,-1) + 4F(1,0) + F(1,1)\}\end{aligned}$$

is applied to

$$\int_{-1}^{1}\int_{-1}^{1} F(x,y)\ dxdy.$$

Describe the 2-variable polynomials that are integrated exactly by this rule. [6 marks]

Why is the product form of $\mathbf{S}f$ unsuitable for integrating over a hypercube in 20 dimensions? Name a better method for 20 dimensions on a sequential machine, given that high accuracy is not required. [3 marks]